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Suppression of Bending–Torsion Flutter Through Displacement-Dependent Dry Friction Damping

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Flutter is an important problem in modern technology engines. Despite significant research in the area of turbomachinery aeroelasticity, flutter instabilities still occur. The flutter suppression potential of dry friction damping with displacement-dependent friction forces is investigated. Bending–torsion coupling, aerodynamic coupling, and realistic aerodynamic damping terms are all considered. The total aeroelastic system is studied under compressible flow conditions and is modeled by a set of first-order differential equations in state-space form. Through time integration studies, it is found that the nondimensional flutter speed can be significantly increased by including displacement-dependent friction forces and by moving the location of the dry friction damper. This result strongly suggests that it may be possible to design the geometry and location of frictional interfaces on airfoils so that damping may be enhanced and flutter can be better controlled. Possible future applications might be in the area of turbine or fan and compressor blade systems.

Introduction

THE stabilizing potential of displacement-dependent dry friction damping on coupled bending–torsion flutter is addressed. Flutter is an important problem in modern technology engines. Despite significant research in the area of turbomachinery aeroelasticity, flutter instabilities still occur in current turbines or fan and compressor blade systems.[1–5] Because of the high temperatures and high rotation speeds under which gas turbines operate, effective means of flutter suppression are not easily implemented. One technique that has been explored is that of adding dry friction energy dissipation to turbine and fan blade systems.[6–9] A shortcoming of this approach is that the effective damping supplied by dry friction is known to decrease with increasing slip displacement. Hence, although dry friction may be effective in suppressing low-amplitude flutter, its performance falls off for large disturbances.[10,11] The deficiency stems from the stabilizing friction forces being constant in magnitude regardless of the flutter amplitude.

Recently, the characteristics of systems with displacement-dependent friction forces have been investigated.[6,12–14] It has been found that the damping of such systems can be made to resemble linear structural damping when the forces normal to the frictional interface (hereafter referred to as normal forces) are allowed to grow with slip displacement. Like other structural systems with linear material damping, the energy loss per cycle can be made to grow like the square of the vibratory displacement amplitude rather than linearly with amplitude as in the case of frictionally damped systems with constant normal forces. Furthermore, dry friction is very well suited to high-temperature applications.[7] This suggests that displacement-dependent dry friction may be an effective means of flutter suppression in turbine blade and fan blade applications.

It is generally accepted that flutter is essentially a single-mode phenomenon, with bending–torsion coupling having an important impact on the flutter boundaries.[15] Even so, coupled bending–torsion flutter has received little attention in the literature. In fact, prior to Bendiksen and Friedmann's work,[15] no results were published comparing single-degree-of-freedom (SDOF) and coupled bending–torsion stability boundaries. Carta[16] analyzed the problem of coupled blade–disk–shroud flutter in 1967. In 1982, Kaza and Kielb[17] published a paper on the effects of mistuning on bending–torsion flutter as applied to the response of a cascade in incompressible flow. Most relevant to the current work is a paper published by Sinha et al.[18] that studied the direct effect of friction damping on torsional blade flutter. It should be noted, however, that Sinha et al. considered only torsional blade flutter without coupled bending.

This paper investigates the flutter suppression potential of dry friction with displacement-dependent normal forces. Bending–torsion coupling, aerodynamic coupling, and realistic aerodynamic damping terms are all considered. Incorporating these phenomena provides a realistic method for assessing how effective displacement-dependent dry friction damping may be in an actual application. The remainder of the paper is organized as follows: The next section describes the aerodynamic model to be used. The third section is a development of the structural model. Results and a discussion are provided in the fourth section. Conclusions are presented in the final section.

Aerodynamic Model

This investigation uses a linear, state-space, time-domain model of unsteady aerodynamics in a compressible flow for the flutter analysis. This model was developed by Leishman and Crouse[19] and Crouse and Leishman[20] and predicts unsteady lift, moment, and drag acting on a two-dimensional airfoil for subsonic and transonic flow. Note that there is some concern and debate over whether a linear model such as in Refs. 19 and 20 is entirely accurate in the upper transonic regime. At high angles of attack and higher Mach numbers, the role of supercritical flow phenomena and shock waves render this linear model increasingly invalid.[21] However, for the purposes of this research and the basic nature of this study, it is felt that this model is adequate in examining the effects of displacement-dependent dry friction on typical airfoils.

Starting from suitable generalizations of indicial aerodynamic functions, Leishman and Crouse represent unsteady loads due to an arbitrary forcing in a set of first-order differential equations in state-space form. The inputs to this aerodynamic system are angle of attack and pitch rate. The outputs are unsteady lift and moment. The Leishman/Crouse model has been validated against various experimental data and computational fluid dynamic solutions for harmonic pitch oscillations up to Mach numbers of 0.875 (Ref. 22). Recently,

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Leishman and Crouse's aerodynamic model was combined with a structural system having hardening-spring-type nonlinearities.[23] The stability of the resulting nonlinear system was studied using an averaging method. In the present work, we take advantage of the time-domain aerodynamic representation to investigate the stability of the nonlinear aeroelastic system using a time-marching technique.

The state-space formulation of the aerodynamic model allows it to be easily coupled to structural equations of typical two-DOF airfoil sections. Leishman and Crouse did so as an example with a NACA 64A006 airfoil for which experimental data were available. Because their combined aeroelastic system was linear, stability could be determined by either eigenanalysis or direct time integration.

Leishman and Crouse modeled aerodynamic loads under attached flow conditions while undergoing arbitrary motion using the indicial response method in conjunction with linear superposition. In addition, they placed no constraints on the formulation of the blade structural response. This type of indicial response approach is typically used in the analysis of both fixed-wing and rotary-wing aeroelasticity problems. The classic case is the use of the Wagner function (see, for example, Ref. 24) for an airfoil operating in incompressible flow for a step change in angle of attack. The Wagner function, however, has limited practical utility at higher Mach numbers, where flow compressibility effects become important. It is well known that flutter problems are more likely to occur near transonic Mach numbers. In this regard, the Leishman/Crouse model is more appropriate for flutter analysis.

It should be noted that Friedmann and Venkatesan,[25] Venkatesan and Friedmann,[26] and Dinyavari and Friedmann[27–29] have also developed and implemented state-space representations of unsteady aerodynamic effects, but they restricted their work to incompressible flow using the classical Wagner function. Similar work was also done by Edwards et al.[30] but again was limited to incompressible flow by reformulating the Wagner function.

Following the development in Ref. 19, the general aerodynamic system is defined by eight states in the form

$$\dot{\boldsymbol{x}} = [A]\boldsymbol{x} + [B]\begin{Bmatrix}\alpha\\ q\end{Bmatrix} \tag{1}$$

with the output equations

$$\begin{Bmatrix}C_N\\ C_M\end{Bmatrix} = [C]\boldsymbol{x} + [D]\begin{Bmatrix}\alpha\\ q\end{Bmatrix} \tag{2}$$

where $\boldsymbol{x}$ is an 8×1 vector of aerodynamic state variables, α is the angle of attack, q is the nondimensional pitch rate, C_N is the normal force coefficient, and C_M is the pitching moment coefficient about the one-quarter chord. Further details of the state-space formulation are given by Leishman and Crouse.[19]

Prior to combining the aerodynamic model with the structural model described in the following section, it was compared extensively to the published results of Leishman and Crouse. In particular, the indicial functions were plotted at various Mach numbers for step changes in angle of attack and pitch rate about the quarter chord. These plots agreed with those published by Leishman.[22]

Structural Model

One may easily couple the structural equations of motion of an airfoil with the aerodynamic equations from the preceding section. The structural model used in this study is shown in Fig. 1. It is an equivalent two-dimensional cross-sectional representation of a dry friction damped fixed-wing airfoil. Research and experience have confirmed that, for purposes of flutter prediction, this type of section is representative of a straight airfoil of large span; the geometric and inertial properties of the section are selected to match those of the airfoil's three-quarters station.[31] Aerodynamic operating conditions are taken at the same location. The parameters used for this model are given in Table 1. These beam parameter values were chosen to approximate an actual turbomachinery fan blade; however, cascade effects are neglected, and the emphasis is on qualitative trends in the dynamic behavior of the generic system. Note that certain sections of this paper vary these baseline parameter values to examine their effects on the dynamic response.

In Fig. 1, bending stiffness and torsional rigidity are referred to the elastic axis and can be represented by springs of stiffnesses K_h and K_θ, respectively. The airfoil is assumed to be rigid in the chordwise direction. The coupling between bending and torsion due to pretwist, shrouds, and rotation of the rotor can be modeled through the offset distance between the center of gravity and elastic axis x_{cg} (positive aft). The centrifugal stiffening effects due to rotation can be included in the bending and torsional spring constants.

The dry friction damper element shown in Fig. 1 is similar to that examined previously by Whiteman and Ferri.[14] The friction force F_f is assumed to be given by Coulomb's law:

$$F_f = \mu N_x \operatorname{sgn}(\dot{w}_d) \tag{3}$$

where $\dot{w}_d$ is the damper velocity, N_x is the normal force pressing the sliding interface together, and μ is the (sliding) coefficient of friction, which is assumed to be equal to the static coefficient of friction in this study. The signum (sgn) term is defined to be equal to $+1$, -1, or 0, depending on whether the argument is greater than zero, less than zero, or 0, respectively. (The situation where the

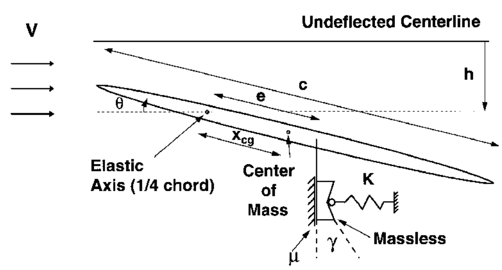

Fig. 1 Structural model.

Table 1 Baseline parameter values

Item	Symbol	Value	Units
Airfoil chord	c	0.1	m
Semichord, $c/2$	b	0.05	m
Airfoil mass ratio	$\mu_m = m/\rho_{\text{air}}\pi b^2$	65.85	——
Mass per unit span	m	0.6361	kg/m
Polar mass moment of inertia about elastic axis per unit span	I_θ	0.398×10^{-3}	kg-m
Radius of gyration about elastic axis (nondimensional distance in semichords)	$r_\theta = \sqrt{I_\theta/mb^2}$	0.5	——
Static mass moment	$S_\theta = mbx_{cg}$	Varies	kg
Center of gravity–elastic axis offset (nondimensional distance in semichords)	x_{cg}	Varies	——
Frequency ratio, bending to torsion	ω_h/ω_θ	Varies	——
Spring stiffness associated with dry friction normal force	K	1000	N/m
Coefficient of friction	μ	0.5	——
Structural damping in plunging and pitching	g_h, g_θ	Varies	——
Preload force	N_0	5	N

frictional interface sticks and $\dot{w}_d = 0$ for finite lengths of time is treated subsequently.)

Because of the inclined profile or ramp angle γ, the normal force N_x is not constant but increases with damper displacement w_d:

$$N_x = N_0 + K|w_d|\tan\gamma \tag{4}$$

where K is the stiffness of spring oriented normal to the sliding direction and N_0 accounts for any preload in the spring ($N_0 \geq 0$).

In addition to the friction force, there is a force N_y that results from the in-plane component of the force exerted on the ramp surface by the spring/roller:

$$N_y = N_x \tan\gamma \operatorname{sgn}(w_d) \tag{5}$$

Though perhaps negligible for small ramp angles, the in-plane force component was found to have a significant effect on the vibratory characteristics as γ increases.[14] The total force F_d exerted by the damper on the airfoil is given by

$$F_d = \mu N_x \operatorname{sgn}(\dot{w}_d) + N_y \tag{6}$$

Finally, the damper displacement and velocity, which are defined to be positive downward, are related to the airfoil motion as follows:

$$w_d = h + e\theta \tag{7}$$

$$\dot{w}_d = \dot{h} + e\dot{\theta} \tag{8}$$

where h is the plunge (bending) displacement (positive downward), θ is the pitch angle (positive nose up), and e is the position of the dry friction damper in relation to the elastic axis (positive aft).

The coupled bending–torsion equations follow directly from the application of Newton/Euler principles or through Lagrange's equations:

$$m\ddot{h} + S_\theta\ddot{\theta} + g_h\dot{h} + m\omega_h^2 h = -L - \mu N_x \operatorname{sgn}(\dot{h} + e\dot{\theta}) - N_y \tag{9}$$

$$S_\theta\ddot{h} + I_\theta\ddot{\theta} + g_\theta\dot{\theta} + I_\theta\omega_\theta^2\theta = M_{ac} - e[\mu N_x \operatorname{sgn}(\dot{h} + e\dot{\theta}) + N_y] \tag{10}$$

where m is the mass per unit span, S_θ is the static mass moment of inertia, and I_θ is the polar mass moment of inertia about the elastic axis. The nonfrictional damping in plunging and pitching is g_h and g_θ, respectively. The uncoupled bending and torsion frequencies are $\omega_h = \sqrt{(K_h/m)}$ and $\omega_\theta = \sqrt{(K_\theta/I_\theta)}$. The dimensional lift and moment about the aerodynamic center are given by

$$L = \tfrac{1}{2}\rho_{air}V^2 c C_N \tag{11}$$

$$M_{ac} = \tfrac{1}{2}\rho_{air}V^2 c^2 C_M \tag{12}$$

where ρ_{air} is the air density, V is the freestream velocity, and c is the airfoil chord. Note that the normal force coefficient C_N is assumed to be approximately equal to the lift coefficient C_L. Because an airfoil has high in-plane stiffness for fixed-wing problems, the chord (in-plane) force component rarely participates in the aeroelastic problem, further justifying this approximation.

Equations (9) and (10) may be written in state-space form and coupled with the Leishman/Crouse aerodynamic model, yielding a total aeroelastic system consisting of 12 states. The structural equations contain discontinuous terms because of the signum function used in the friction model. These discontinuities present computational difficulties during numerical integration. In particular, when sticking occurs, the discontinuity in the friction law causes the friction force to switch directions, excessively at times, with every time step. This can lead to very high computational times and a significant loss of accuracy. To avoid these problems, a sticking condition is checked each time the slip velocity $\dot{w}_d$ crosses zero. If the friction interface is stuck, a set of linear equations is used, representing the airfoil's dynamics during sticking. The linear structural equations are

$$m\ddot{h} + S_\theta\ddot{\theta} + g_h\dot{h} + m\omega_h^2 h = -L + \beta \tag{13}$$

$$S_\theta\ddot{h} + I_\theta\ddot{\theta} + g_\theta\dot{\theta} + I_\theta\omega_\theta^2\theta = M_{ac} + e\beta \tag{14}$$

where β is the force of constraint at the stuck damper interface. In addition to these two differential equations, there exists an algebraic constraint relation

$$w_d = h + e\theta = \text{const} \tag{15}$$

The linear structural equations (13–15) can again be coupled with the Leishman/Crouse aerodynamic model. The constraint force β can then be eliminated to provide a reduced-order set of equations to represent the total aeroelastic system. Theoretical development of this reduced-order system follows directly from the application of component mode synthesis and is very similar to that used by Whiteman and Ferri[32] in an investigation of a multi-DOF beam system. Reference 32 also contains a further discussion of how the sticking/slipping criterion is implemented in a time integration algorithm.

Results and Discussion

The total aeroelastic system equations developed in the preceding section may now be analyzed to predict flutter conditions for the two-DOF cross section in Fig. 1. Because the governing structural equations are highly nonlinear, this is carried out by numerical integration to determine stability or instability.

The flutter speed of the blade-like structure can be computed at various Mach numbers. The mass and stiffness parameters in Table 1 were used along with the specific values of $x_{cg} = 0.05$ and $\omega_h/\omega_\theta = 0.16$. No applied damping was included, i.e., $g_h = g_\theta = 0$.

An initial pitch angle displacement of 1 deg was used to start the integration. (All other aerodynamic and structural initial conditions were set to zero.) The integration was performed using a fourth-order Runge–Kutta time-marching routine. This time-marching procedure can be easily repeated for different speeds and at different Mach numbers. To determine the flutter boundary, the time histories of the torsional and plunge motions were plotted for velocities above and below the critical flutter speed. Below the critical flutter speed, both the bending and torsional motions are positively damped, and the oscillations die out. Above the critical flutter speed, the motion grows in amplitude over time.

Figure 2 is a plot of the nondimensionalized flutter speed vs ramp angle for Mach = 0.85 and a damper offset of $e = c/4$ (midchord). Figure 2 indicates that the flutter speed increases nearly linearly as the ramp angle of the dry friction damper is increased. Note that increasing the ramp angle has the effect of increasing the level of displacement dependence of the friction force. Figure 3 is a similar plot for Mach = 0.85 and a damper offset of $e = c/2$ ($\frac{3}{4}$ chord). As expected, it is noted in Fig. 3 that the nondimensionalized flutter speed increases and the slope of the curve also increases as the dry friction damper becomes more effective when it is placed farther from the elastic axis.

A parametric study was undertaken with the same structure replacing the displacement-dependent dry friction damper with either a linear spring or a linear viscous damper. In both cases, similar results were achieved in which the nondimensionalized flutter speed

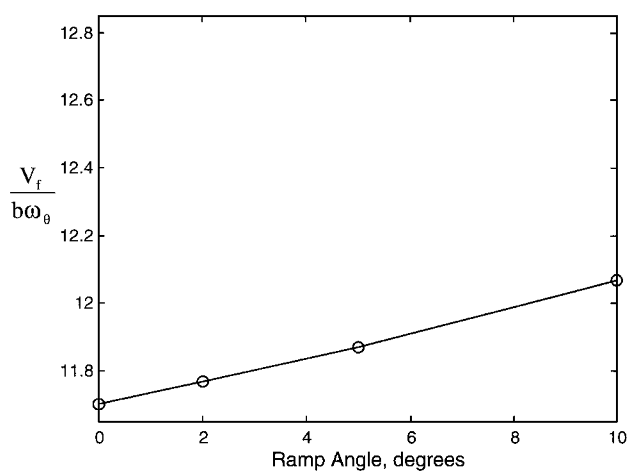

Fig. 2 Nondimensional flutter speed vs ramp angle γ; Mach $M = 0.85$ and damper offset $e = c/4$.

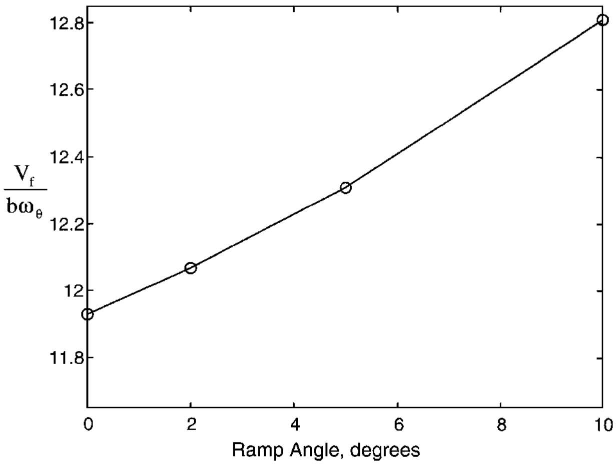

Fig. 3 Nondimensional flutter speed vs ramp angle γ; Mach $M = 0.85$ and damper offset $e = c/2$.

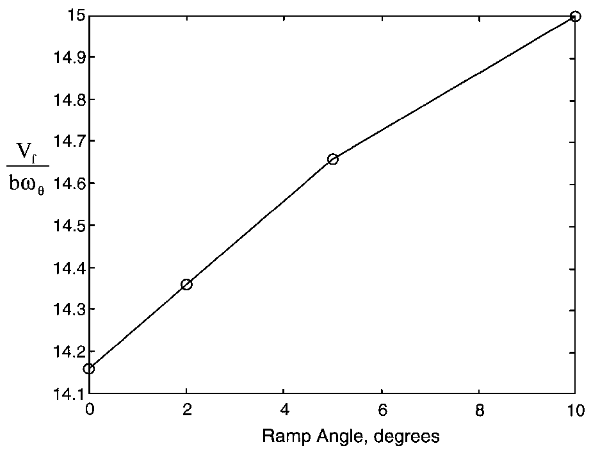

Fig. 4 Nondimensional flutter speed vs ramp angle γ; Mach $M = 0.5$ and damper offset $e = c/4$.

increased with both increasing stiffness and damping. In addition, these linear elements were more effective in increasing the flutter speed as they were placed farther from the elastic axis. Results from the displacement-dependent dry friction damper case indicate that it produces the benefit of both increasing the stiffness at that location and providing a linear structural-like damping effect.

As already mentioned, the results in Figs. 2 and 3 were at a Mach number of 0.85. This is in the transonic flow range in which flutter often becomes a problem. To validate the conclusions drawn about the airfoil's response with the attached displacement-dependent dry friction damper, additional studies were conducted at a subsonic flow range of Mach = 0.5. Figure 4 is a typical plot of these results for a damper offset, $e = c/4$ (midchord). In general, as expected, the nondimensionalized flutter speed at this reduced flow range is much higher. The important conclusion, however, is that the displacement-dependent dry friction damper still increases the flutter boundary as the ramp angle is increased.

Because the present system is nonlinear, it is natural to inquire about the influence of initial conditions. In an earlier work by Ferri and Dowell,[10] which considered only constant normal loads, it was found that initial conditions had a profound effect on the stability of the response. Furthermore, the study showed complicated domains of attraction for various types of ensuing motion.

Previous work has shown that the damping contribution of displacement-dependent dry friction is linear structural-like in nature.[14,32,33] It has also been observed that, in examining stability effects, systems with displacement-dependent dry friction are relatively insensitive to changes in initial conditions.[33] As mentioned earlier, this was in contrast to earlier work by Ferri and Dowell[10] on similar models in which the normal force at the dry friction damper remained constant.

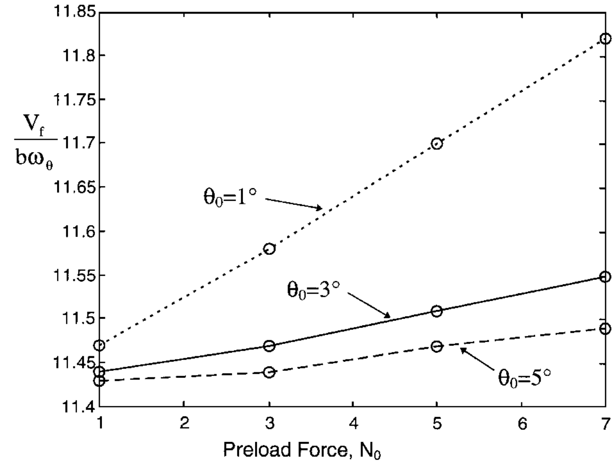

Fig. 5 Nondimensional flutter speed vs preload force N_0; Mach $M = 0.85$, damper offset $e = c/4$, and ramp angle $\gamma = 0$, at various initial conditions.

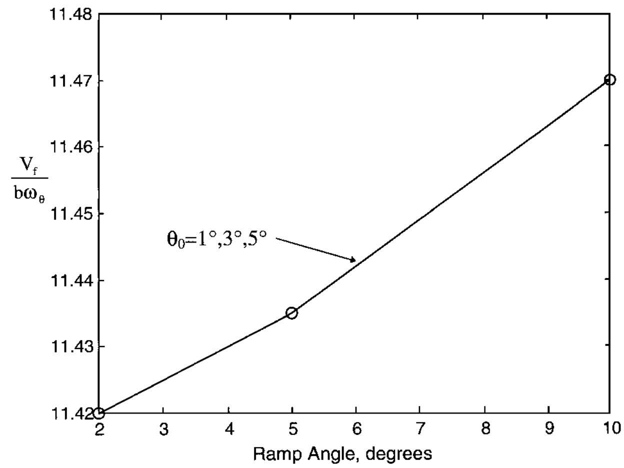

Fig. 6 Nondimensional flutter speed vs ramp angle γ; Mach $M = 0.85$, damper offset $e = c/4$, and preload force $N_0 = 5$ N, at various initial conditions.

To confirm these observations for the present system, a study was conducted in which only the preload force was included. This corresponds to the case of a friction force of constant magnitude. Figure 5 is a plot of the results for $M = 0.85$ and the damper offset at the midchord. Various initial pitch angle rotations θ_0 were used to start the integration. Figure 5 clearly shows that increasing the normal force at the frictional interface increases the nondimensionalized flutter speed. Increasing the preload force N_0 beyond a value of $7N$ continues to increase the nondimensionalized flutter speed; however, sticking becomes more and more prevalent. It is also clear from Fig. 5 that the system is quite sensitive to the initial conditions. Whereas the nondimensionalized flutter speed never drops below the value obtained without the damper present, the system is less stable for larger initial conditions and flutter occurs at a lower nondimensionalized speed.

Figure 6 is a similar plot. The preload force, $N_0 = 5N$, is included in this case, however, and only the ramp angle is changed to vary the effects of displacement dependence. As observed earlier, increasing the ramp angle increases the nondimensionalized flutter speed. The other critical observation is that changing the initial conditions has no discernible effect on the stability condition of the system. The nondimensionalized flutter speed at which flutter occurs is unchanged regardless of the initial conditions or amplitude of the response. This result further confirms that displacement-dependent dry friction alone is much like linear structural damping and much more efficient in controlling flutter than systems with dry friction and constant normal loads.

Conclusions

The unsteady aerodynamic behavior of a two-dimensional airfoil with an attached displacement-dependent dry friction damper is studied under compressible flow conditions. Using an aerodynamic model developed by Leishman and Crouse,[19] the total aeroelastic system is described by a set of first-order ordinary differential equations in state-space form. The aerodynamic modeling has been previously validated against a selection of experimental and computational fluid dynamic data for oscillating airfoils by Leishman and Crouse.

The state-space aerodynamic model is combined with a dry friction damped structural model. The resulting aeroelastic system is integrated in time using a fourth-order Runge–Kutta routine. The simulated behavior is monitored for growing or diminishing oscillations to determine the flutter boundary. A key finding was that the nondimensionalized flutter speed could be significantly increased by increasing the ramp angle of the dry friction damper and moving its location. In addition, with the inclusion of displacement-dependent dry friction, the stability of the system was insensitive to changes in initial conditions, and the damping had an effect similar to that normally attributed to linear structural damping. These results strongly suggest that it may be possible to design the geometry and location of frictional interfaces on airfoils so that damping may be enhanced and flutter can be better controlled. Possible future applications might be in the area of turbine or fan and compressor blade systems.

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